

## Intervalence band Faraday effect in germanium\*

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Using the theory of interband Faraday effect in solids given by Roth, the theoretical values of the transmission Faraday rotation due to the intervalence band transitions at the  $\Gamma$  point are calculated. To simplify two integrals we introduce two parameters which play the same role as width in dispersion theory. After fitting these two parameters with the experiment the theoretical calculations agree very well with the experimental observations in Ge.

### 1. INTRODUCTION

The intervalence band Faraday rotation at the  $\Gamma$  point was first observed in  $p$ -type germanium by Walton & Moss (1961), germanium being one of the most favourable material for observing this effect. These authors gave a qualitative explanation of the effect on the basis of classical theory. In the present paper we propose to report the results of the theoretical investigation of the Faraday effect caused by intervalence band transitions using the general result of interband Faraday rotation (Roth 1964, Ray 1976) and its comparison with experimental results for germanium.

The general result in terms of the conductivity tensor  $\sigma_{xy}^{(i,b)}$  to the first order in magnetic field due to direct interband transitions (Roth 1964) is

$$\sigma_{xy}^{(i,b)} = -ie^2 \hbar c_{\alpha\beta} \epsilon_{\gamma\delta} \sum'_{nn'n''k} [\Pi_{nn'}^{\alpha} (x^{\gamma} (\Pi^{\delta} + v^{\delta}) + S_{\gamma\delta})_{n'n''} \Pi_{n''n}^{\beta} - (\Pi x^{\gamma} + i\delta_{\alpha\gamma}/m)_{nn'} (\Pi^{\delta} + v^{\delta})_{n'n''} \Pi_{n''n}^{\beta}] \frac{1}{E_{n'n''}^2} \left\{ \frac{f_{nn'}}{E_{nn'}^2 - w^2} - \frac{f_{n'n''}}{E_{n'n''}^2 - w^2} \right\} \quad \dots (1)$$

where the prime on the sum excludes the case where the three energies are equal. The symbols having the same meanings as in Roth (1964).  $\sigma_{xy}^{(i,b)}$  consists essentially of two terms, the first in the square bracket is the contribution from Zeeman interaction  $H'$  of a band electron and the second is the contribution from the change in the matrix element  $\pi'$  due to the presence of the magnetic field.

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As is usual we represent by  $V_1$  and  $V_2$  the doubly degenerate valence bands degenerate at the  $\Gamma$  point,  $V_3$  the spin orbit split off valence band, and  $C$  the conduction band. In the  $p$ -type material at normal temperatures the upper two valence bands  $V_1$  and  $V_2$  are occupied near  $k = 0$  by the so-called heavy and light holes respectively, while the split off band  $V_3$  is fully occupied by electrons. The intervalence band Faraday rotation in the  $p$ -type semiconductor is due to the dispersion in the presence of the magnetic field associated with the intervalence band transitions of the type  $V_1 \leftrightarrow V_2$ ,  $V_2 \leftrightarrow V_3$  and  $V_1 \leftrightarrow V_3$ . In the next section we calculate the contributions to the conductivity tensor  $\sigma_{ij}^{(a,b)}$  from the above three transitions. It is seen that there is no contribution from the transition  $V_1 \leftrightarrow V_2$  due to the fact that the valence to conduction band energy gap  $E_g$  is very much larger than  $\Delta_{12}$ , the energy gap between the valence bands  $V_1$  and  $V_2$  at the  $\Gamma$  point.

## 2. THE CONDUCTIVITY TENSOR DUE TO THE INTERVALENCE BAND TRANSITIONS

The form of the basis functions (Roth 1964, Ray 1976) at  $k = 0$  and their transformation properties gives at  $k = 0$

$$\begin{aligned} \Pi^a_{V_i V_j} &= 0, & \text{for all } i, j \\ \Pi^a_{C V_j} &\neq 0, & \text{for all } j \end{aligned} \quad (2)$$

When we consider intervalence band transitions the subscripts  $n, n'$  in  $f_{nn'}$  refer to the valence bands  $V_1, V_2$  or  $V_3$ . As we have done previously (Ray 1976) in the case of interband Faraday effect here also we will evaluate the matrix elements at the band edge only. From eqs (1) and (2) it is evident that we get the non-zero contribution to the conductivity tensor only when the subscript  $n''$  which is not associated with  $f_{n''}$  refers to the conduction band  $C$ . Therefore, when we consider the first term in the curly bracket of eq (1)  $n$  and  $n'$  go over to  $V_1, V_2$  or  $V_3$  and  $n''$  to  $C$ , in the second term  $n$  and  $n''$  go over to  $V_1, V_2$  or  $V_3$  and  $n'$  to  $C$ . Using eq. (2) we see that when we take  $n'' = C$ ,  $\Pi^a_{nn'}$  in the first term of the square bracket goes to zero; when  $n' = C$ ,  $\Pi^b_{n''n}$  in the first term goes to zero. As for the second term in the square bracket when we take  $n' = C$ ,  $\Pi^b_{n''n}$  in the second term goes to zero. But when  $n'' = C$ , the second term is non zero. Since we evaluate the matrix elements only at the band edge we shall assume that the only contribution to the intervalence band Faraday rotation coming from this term is important and neglect the contributions coming from other terms. Thus the important term in the present case is coming from the change in matrix element due to the presence of the magnetic field in eq (1), which was neglected by Roth (1964) in her detailed calculation of interband Faraday rotation for individual semiconductors and was shown to be important

in those cases by Ray (1976). Evaluating the matrix elements at the band edge  $\sigma_{xy}^{(vib)}$  for the intervalence band Faraday effect may be written from eq. (1) as

$$\sigma_{xy}^{(vib)} = ie^2 \hbar \epsilon_{\alpha\beta\gamma\delta} \sum_{\substack{nn' \\ ck}} \left[ \left( \Pi_{\alpha}^{\gamma} x^{\gamma} + i \frac{\delta_{\alpha\gamma}}{m} \right)_{nn'} \right. \\ \left. \times (\Pi_{\delta}^{\beta} + v_{\delta}^{\beta})_{n'c} \Pi_{cn}^{\beta} \right]_{k=0} = \frac{f_{nn'}}{E_{n'c}(E_{nn'}^2 - w^2)} \quad \dots \quad (3)$$

Since we are going to use orthonormal basis therefore,

$$\left( i \frac{\delta_{\alpha\gamma}}{m} \right)_{nn'} = \left( i \frac{\delta_{\alpha\gamma}}{m} \right)_{nn} \delta_{nn'} = 0. \quad \dots \quad (4)$$

Using relation (2) we get

$$(\pi_{\alpha}^{\gamma} x^{\gamma})_{nn'} = \sum_c \pi_{nc}^{\alpha} x_{c'n'}^{\gamma} \quad \dots \quad (5)$$

Also from the definition of  $v_{nn'}^{\delta}$  and relation between  $x_{nn'}^{\delta}$ ,  $\pi_{nn'}^{\delta}$  and  $v_{nn'}^{\delta}$  (Roth 1964), we have

$$v_{n'c}^{\delta} = 0 \\ \text{and} \quad \pi_{cn}^{\beta} = -E_{cn} x_{cn}^{\beta} / i \quad \dots \quad (6)$$

The matrix elements in eq. (3) above may be written as

$$\hbar \epsilon_{\beta\alpha} x_{cn}^{\beta} \Pi_{nc}^{\alpha} = (\mathbf{h} \cdot \mathbf{g}_c'(n) \mathbf{S}) / m \\ \hbar \epsilon_{\gamma\delta} x_{c'n'}^{\gamma} \Pi_{n'c'}^{\delta} = (\mathbf{h} \cdot \mathbf{g}_c'(n') \mathbf{S}) / m \quad \dots \quad (7)$$

where  $g_c'(n)$  is the contribution to the conduction band  $g$ -factor from the valence band  $n$  only, and  $S$  is the conduction band pseudo spin. Utilizing eq. (7)  $\sigma_{xy}^{(vib)}$  is given by

$$\sigma_{xy}^{(vib)} = \frac{e^2}{m^2 \hbar} \sum_{knn'} \sum_{ij} \left[ (\mathbf{h} \cdot \mathbf{g}_c'(n) \mathbf{S}_i) (\mathbf{h} \cdot \mathbf{g}_c'(n') \mathbf{S}_j) E_{cn} \right]_{k=0} \frac{f_{nn'}}{E_{cn'}(E_{nn'}^2 - w^2)} \quad (8)$$

where  $i, j$  denote pseudo spin states

From eq. (7)  $g_c'(n)$  may be written as

$$(\mathbf{g}_c'(n) \mathbf{S}) = m(\Pi_{cn} \times \Pi_{nc'}) / i E_{cn} \quad \dots \quad (9)$$

Now let us define  $\mathbf{g}_c''(n)$  by

$$(\mathbf{g}_c''(n) \mathbf{S}) = E_{cn} (\mathbf{g}_c'(n) \mathbf{S}) \\ = \frac{m}{i} (\boldsymbol{\pi}_{cn} \times \boldsymbol{\pi}_{nc'}) \quad \dots \quad (10)$$

so that it can easily be shown that

$$\sum_{ij} (\mathbf{h} \mathbf{g}_c''(n) \mathbf{S}_{ij})(\mathbf{h} \mathbf{g}_c''(n') \mathbf{S}_{ji}) \\ = -\frac{1}{2} \sum_{\alpha} h_{\alpha}^2 g_c''^{\alpha\alpha}(n) g_c''^{\alpha\alpha}(n') \quad \dots (11)$$

Using eqs (10), (11) and summing the band indices,  $n$  and  $n'$  over  $V_1$ ,  $V_2$  and  $V_3$  we get,

$$\sigma_{xy}^{(vib)} = \frac{e^2}{2m^2\hbar} \sum_{\alpha} \left[ \frac{h_{\alpha}^2 g_c''^{\alpha\alpha}(1) g_c''^{\alpha\alpha}(3)}{E_{c3}^2} \left( \sum_k \frac{f_{13}}{E_{13}^2 - w^2} \right) \right. \\ + \frac{h_{\alpha}^2 g_c''^{\alpha\alpha}(3) g_c''^{\alpha\alpha}(1)}{E_{c1}^2} \left( \sum_k \frac{f_{31}}{E_{13}^2 - w^2} \right) \\ + \frac{h_{\alpha}^2 g_c''^{\alpha\alpha}(2) g_c''^{\alpha\alpha}(3)}{E_{c3}^2} \left( \sum_k \frac{f_{23}}{E_{23}^2 - w^2} \right) \\ \left. + \frac{h_{\alpha}^2 g_c''^{\alpha\alpha}(3) g_c''^{\alpha\alpha}(2)}{E_{c2}^2} \left( \sum_k \frac{f_{32}}{E_{23}^2 - w^2} \right) \right] \quad \dots (12)$$

The first two terms give the contribution from the transition  $V_1 \longleftrightarrow V_3$  and the last two terms from the transition  $V_2 \longleftrightarrow V_3$ . The contribution from the transition  $V_1 \longleftrightarrow V_2$  gives two terms which are similar to the terms from the transitions  $V_1 \longleftrightarrow V_3$  and  $V_2 \longleftrightarrow V_3$  and since  $E_{c1} \approx E_{c2}$ , and  $f_{12} = -f_{21}$  the two terms cancel each other. Thus only the transitions  $V_1 \longleftrightarrow V_3$  and  $V_2 \longleftrightarrow V_3$  contribute to the conductivity tensor.

We first calculate the matrix elements  $D_c'' c'^{\alpha\beta}(n)$  defined by

$$D_c'' c'^{\alpha\beta}(n) = m^2 (\Pi_{cn} \alpha \Pi_{nc} \beta) \quad \dots (13)$$

and then use the relation

$$g_c''^{\alpha\beta}(n) S_{\beta} = \epsilon_{\alpha\beta\gamma} D_c'' c'^{\beta\gamma}(n) / i\hbar m \quad \dots (14)$$

to evaluate  $g_c''^{\alpha\alpha}(n)$ , where  $\epsilon_{\alpha\beta\gamma}$  is the antisymmetric third rank tensor. To find the matrix elements  $D_c'' c'^{\alpha\beta}(n)$  as in Roth (1964) and Ray (1976) we use the basis functions in which the Hamiltonian is diagonal. Thus the basis functions corresponding to the valence band  $V_1$  are

$$\phi_1 = (i/\sqrt{2}) \mathbf{R}(\mathbf{s} + i\mathbf{t}) \alpha' \\ \phi_1' = (i/\sqrt{2}) \mathbf{R}(\mathbf{s} - i\mathbf{t}) \beta'$$

corresponding to the valence band  $V_2$  are

$$\phi_2 = (i/\sqrt{6})\mathbf{R}[(\mathbf{s} + i\mathbf{t})\beta' - 2\mathbf{r}\alpha']$$

$$\phi_2' = (i/\sqrt{6})\mathbf{R}[-(\mathbf{s} - i\mathbf{t}) - 2\mathbf{r}\beta']$$

and corresponding to the valence band  $V_3$  are

$$\phi_3 = (i/\sqrt{3})\mathbf{R}[-(\mathbf{s} + i\mathbf{t})\beta' - \mathbf{r}\alpha']$$

$$\phi_3' = (i/\sqrt{3})\mathbf{R}[(\mathbf{s} - i\mathbf{t})\alpha' - \mathbf{r}\beta']$$

where  $\mathbf{R}$ ,  $\mathbf{s}$ ,  $\mathbf{t}$ , and  $\mathbf{r}$  are defined as in Ray (1976) and  $\alpha'$ ,  $\beta'$  the spin states quantized in  $z$  direction. Here we take the basis functions for the conduction band  $c$ , slightly different, namely

$$\phi_c = S\sigma$$

$$\phi_{c'} = S\sigma'$$

where  $\sigma$  and  $\sigma'$  are conduction band spin states quantized in any arbitrary direction

Using the above basis functions and the vector components in the directions  $s$ ,  $t$ , and  $r$  the matrix elements  $D_{cc'}^{\alpha\beta}(n)$  which we need to evaluate  $g_c^{\alpha\beta}(n)$  are obtained. Then using eq. (14) the non-zero components of the tensor  $g_c^{\alpha\beta}(n)$  are given by

$$\left. \begin{aligned} g_c^{\alpha rr}(1) &= -2P^2/m \\ g_c^{\alpha rr}(2) &= 2P^2/3m \\ g_c^{\alpha ss}(2) &= -4P^2/3m - g_c^{\alpha tt}(2) \\ g_c^{\alpha rr}(3) &= -4P^2/3m - g_c^{\alpha ss}(3) - g_c^{\alpha tt}(3) \end{aligned} \right\} \quad \dots \quad (15)$$

where

$$\begin{aligned} P &= (S|p^x|\mathbf{R}\mathbf{s}) - (S|p^x|\mathbf{R}\mathbf{t}) \\ &= (S|p^x|\mathbf{R}\mathbf{r}) = (S|p^x|\mathbf{X}) \end{aligned}$$

and is related to the parameter  $f$  defined in Roth (1964) by

$$P^2/mE_g = f/2 \quad \dots \quad (16)$$

Utilizing eqs (15) and (16) and the fact that  $f_{ij} = -f_{ji}$ , eq. (12) reduces to

$$\begin{aligned} \sigma_{xy}^{(vib)} &= -\frac{e^2 f^2 H}{18m^2 c} \left[ \frac{\Delta_{13}(2E_g + \Delta_{13})}{(E_g + \Delta_{13})^2} \left( \sum_k \frac{f_{13}}{(E_{13}^2 - \hbar^2 k^2)} \right) \right. \\ &\quad \left. + \frac{\Delta_{23}(2E_g + \Delta_{23})}{(E_g + \Delta_{23})^2} \left( \sum_k \frac{f_{23}}{(E_{23}^2 - \hbar^2 k^2)} \right) \right] \quad \dots \quad (17) \end{aligned}$$

where  $\Delta_{ij}$  is the energy gap between the valence bands  $V_i$  and  $V_j$  at the  $\Gamma$  point.

To evaluate the sum over  $k$  we first note that

$$f_{n_h} = 1 - f_n \quad \dots \quad (18)$$

where  $f_{n_h}$  is the Fermi function for holes in the  $n$ -th band. If  $g_{n_h}(E)$  is the density of state function for holes in the  $n$ -th band then the number density  $N_{n_h}$  of holes in the  $n$ -th band is given by

$$N_{n_h} = \int g_{n_h}(E) f_{n_h}(E) dE \quad \dots \quad (19)$$

We suppose that the holes near the top of the valence band behave as free particles with effective mass  $m_{n_h}$  then

$$g_{n_h}(E) = \frac{1}{4\pi^2} (2m_{n_h})^{3/2} (-E)^{1/2} \quad \dots \quad (20)$$

the energy being positive upward from the top of the valence band. Let us now write  $f_{13}$  as

$$f_{13} = (1 - f_{13}) - 1 = -f_{13} \quad \dots \quad (21)$$

and

$$E_1 = -\frac{k^2}{2m_1}, \quad E_3 = -\Delta_{13} - k^2/2m_3$$

so that,

$$E_{13} = \Delta_{13} + k^2/2\mu_{31}$$

where

$$\frac{1}{\mu_{31}} = \frac{1}{m_3} = \frac{1}{m_1} = -\frac{1}{\mu_{13}}$$

Thus we have,

$$\sum_k \frac{f_{13}}{E_{13}^2 - w^2} = \frac{1}{2\pi^2} \int \frac{f_{13} k^2 dk}{\left( \Delta_{13} + \frac{k^2}{2\mu_{31}} \right)^2 - w^2}$$

To simplify the integral we neglect  $k^2$  in the denominator and put in a width so that we get,

$$\sum_k \frac{f_{13}}{E_{13}^2 - w^2} \cong \frac{1}{2\pi^2} \cdot \frac{\Delta_{13}^2 - w^2}{(\Delta_{13}^2 - w^2)^2 + C_{13}^2 w^2} \int_0^\infty f_{13} k^2 dk$$

The effect of the above approximation is that we are replacing  $k$  in the denominator by some sort of average value and then taking the denominator out of the integral. Because the states involved are close to  $k = 0$ , the approximation seems to be not unjustified. We also note that the limit of the values of  $k$  available increases with temperature so that  $C_H$  is larger at high temperature

and from the form of the band structure since the limit of  $k$  is larger in the band  $V_1$  than in the band  $V_2$ .  $C_{13}$  should be greater than  $C_{23}$ . These qualitative predictions are seen to be true experimentally as shown in table 1 for Ge. Using eqs (20) and (21) this integral can be written in the form of eq (19), so that finally we got,

$$\sum_k \frac{f_{13}}{E_{13}^2 - w^2} \cong \frac{(\Delta_{13}^2 - w^2)}{(\Delta_{13}^2 - w^2)^2 + C_{13}^2 w^2} N_{1h}$$

In exactly the same way we get

$$\sum_k \frac{f_{23}}{E_{23}^2 - w^2} \cong - \frac{(\Delta_{23}^2 - w^2)}{(\Delta_{23}^2 - w^2)^2 + C_{23}^2 w^2} N_{2h}$$

$C_{13}$  and  $C_{23}$  are the width associated with the intervalence band transitions  $V_3 \longleftrightarrow V_1$  and  $V_3 \longleftrightarrow V_2$  respectively,  $N_{1h}$  and  $N_{2h}$  are the hole concentrations of valence bands  $V_1$  and  $V_2$  respectively. The part played by the width  $C_{ij}$  here and the width in the optical dispersion are very similar though they have different origin and should not necessarily have the same numerical values.

Applying the above sums over  $k$  eq (17) may be written as

$$\begin{aligned} \sigma_{xy}^{(vib)} = & - \frac{e^3 f^2 H}{18 m^2 c} \left[ \frac{(\Delta_{13}^2 - w^2)}{(E_g + \Delta_{13})^2} \frac{(\Delta_{13}^2 - w^2)}{(\Delta_{13}^2 - w^2)^2 + C_{13}^2 w^2} N_{1h} \right. \\ & \left. + \frac{(\Delta_{23}^2 - w^2)}{(E_g + \Delta_{23})^2} \frac{(\Delta_{23}^2 - w^2)}{(\Delta_{23}^2 - w^2)^2 + C_{23}^2 w^2} N_{2h} \right] \quad \dots (22) \end{aligned}$$

The first term gives the contribution from the transition  $V_1 \longleftrightarrow V_3$  and the second term the contribution from the transition  $V_2 \longleftrightarrow V_3$ .

### 3. THE INTERVALENCE BAND FARADAY ROTATION AND ITS COMPARISON WITH THE EXPERIMENTAL RESULTS OF GERMANIUM

Using the relation between the rotation  $\theta$  and the conductivity  $\sigma_{xy}^{(vib)}$  we have

$$\theta = \theta_0 H \quad \dots (23)$$

where

$$\begin{aligned} \theta_0 = & - \frac{2 \times 10^6 e^3 f^2 \hbar^2}{m^2 c^2 n} \left[ \frac{(\Delta_{13}^2 - w^2)}{(E_g + \Delta_{13})^2} \frac{(\Delta_{13}^2 - w^2)}{(\Delta_{13}^2 - w^2)^2 + C_{13}^2 w^2} N_{1h} \right. \\ & \left. + \frac{(\Delta_{23}^2 - w^2)}{(E_g + \Delta_{23})^2} \frac{(\Delta_{23}^2 - w^2)}{(\Delta_{23}^2 - w^2)^2 + C_{23}^2 w^2} \right] N_{2h} \quad \dots (24) \end{aligned}$$

in deg  $\text{cm}^{-1}$  ( $10^4$  gauss) $^{-1}$  and  $w$ , and energy gaps are in eV. Eq (23) is applicable to semiconductors with Ge-type band structure.

We use eq (23) to calculate the theoretical intervalence band Faraday rotation spectrum of Ge in the three cases which were observed experimentally by Walton & Moss (1961). The constants  $C_{13}$  and  $C_{23}$  which are introduced to simplify integrals are evaluated by fitting the theoretical curve with the experimental curve. The results of our calculations are shown in figures 1 to 3 together with the experimental points. The experimental points are obtained by subtracting off the free hole contribution from the experimental results of Walton & Moss (1961). At 77°K we consider only the contribution from the transition  $V_1 \longleftrightarrow V_3$ , and neglect the contribution from transition  $V_2 \longleftrightarrow V_3$ , which is reasonable on the basis of the experimental evidence of infrared optical absorption spectrum in *p*-type Ge at 77°K observed by Briggs & Fletcher (1953). Their measurements show that the absorption maximum at  $\Delta_{23}$  disappears at the low temperatures. This of course is also evident from the experimental Faraday rotation spectrum which shows a single peak corresponding to the energy  $\Delta_{13}$  at 77°K whereas at 300°K there are two peaks corresponding to the energies  $\Delta_{13}$  and  $\Delta_{21}$ . The values of  $f$ ,  $n$  and  $E_g$  are given Ray (1976) and of the other constants are shown in table 1. In this energy range of the intervalence band gap it is not necessary to consider the contribution coming from the effective gap as discussed in Ray (1976) because the contribution from the effective gap is negligible in comparison with the contribution from the intervalence band gap. Besides the experimental points with which we are comparing our calculated curves are obtained by subtracting the intrinsic contribution which already contains the contribution coming from the effective gap.

Table 1. The values of the constants used to calculate the theoretical curves.

Specimen resistivity at 300°K ( $\Omega$ cm)	Tempera- ture in °K	$N_{13}$ in $\text{cm}^{-3}$ Walton & Moss (1961)	$N_{23}$ in $\text{cm}^{-3}$ Walton & Moss (1961)	$\Delta_{13}$ in eV Briggs & Fletcher (1953)	$\Delta_{23}$ in eV Briggs & Fletcher (1953)	$C_{13}$ in eV	$C_{21}$ in eV
0.0278	77	$3.00 \times 10^{17}$	—	0.330	—	0.0277	—
0.0278	300	$3.80 \times 10^{17}$	$0.266 \times 10^{17}$	0.354	0.275	0.0800	0.02
0.0100	300	$1.35 \times 10^{18}$	$0.945 \times 10^{17}$	0.354	0.275	0.0800	0.03

It is interesting to compare the values of  $C_{ij}$  in table 1 with  $kT$  and the width of optical absorption. The value of  $kT$  at 300°K is 0.0258 eV which compares reasonably well with the values  $C_{ij}$ . If we compare the  $C$ 's with the optical absorption widths  $\Gamma_{ij}$  of the data of Kahn (1955)  $\Gamma_{13} = 0.0037$  eV at 77°K and  $\Gamma_{13} = 0.013$  eV,  $\Gamma_{21} = 0.0074$  eV at 300°K, we find that the  $C$ 's are considerably larger than the  $\Gamma$ 's.



## 1 DISCUSSION

The theoretical spectrum of intervalence band Faraday rotation calculated in section 3 for the three cases of Ge shown in figures 1 to 3 agrees very well with the experimental spectrum of Walton & Moss (1961). The agreement between the calculated curve and the experimental points at 77°K is surprisingly good,

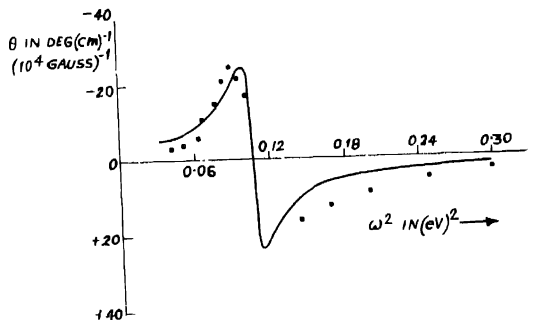


Fig. 1. The intervalence band Faraday rotation as function of  $\omega^2$  in *p*-type Ge of resistivity  $0.0278 \Omega \text{ cm}$  at 77°K. The solid line is the calculated curve and the points are the experimental points of Walton & Moss (1961).

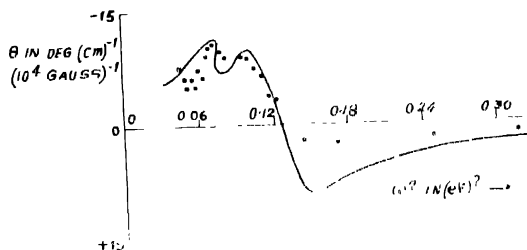


Fig. 2. The intervalence band Faraday rotation as function of  $\omega^2$  in *p*-type Ge of resistivity  $0.0278 \Omega \text{ cm}$  at 300°K. The solid line is the calculated curve and the points are the experimental points of Walton & Moss (1961).

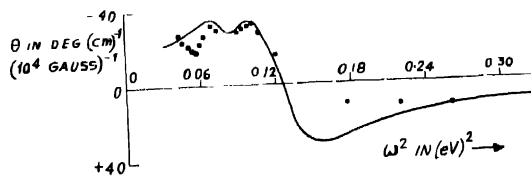


Fig. 3. The intervalence band Faraday rotation as function of  $\omega^2$  in *p*-type Ge of resistivity  $0.0100 \Omega \text{ cm}$  at 300°K. The solid line is the calculated curve and the points are the experimental points of Walton & Moss (1961).

though the experimental peak shows a slightly different position from the theoretical peak, which of course is a matter of finding accurately the value of the gap  $\Delta_{1,3}$ . At 300°K the overall agreement is very good except for the two minor discrepancies. The experimental curves at this temperature seem to show a minimum towards the low frequency side of the intervalence band gaps which the theoretical curves do not show. On the other hand, the theoretical curves show a minimum towards the high frequency side of the intervalence band gaps which is absent in the experimental curves. The causes of these discrepancies are not yet understood. Thus apart from these two minor discrepancies the present model seems to provide a good understanding of the intervalence band Faraday rotation.

Lee & Fan (1966) reported the intervalence band Faraday rotation in *p*-type GaSb, GaAs, InAs and ZnTe but the details of their experimental results are not available. Walton & Mishra (1967) published their experimental results of Faraday rotation in *p*-type GaAs, GaSb and Si. They have also reproduced their previous results on *p*-type Ge. There seems to be a discrepancy of sign of rotation between their previous work and the later work, and they have not mentioned the cause of this discrepancy. They have explained their experimental results on the basis of the theory given by Halpern *et al* (1964). The overall picture of the results of their calculations and our calculations is almost the same apart from the difference of sign, which may simply be due to a different sign convention. For *p*-type GaSb and Si the range of their measurements does not include the frequency of the intervalence band transitions  $V_1 \longleftrightarrow V_3$  and  $V_2 \longleftrightarrow V_3$  and therefore is of no interest to us in the present context. For *p*-type GaAs their measurement show that the intervalence band Faraday rotation spectrum is very similar to Ge at 300°K and can therefore be interpreted very easily on the basis of the present calculations, with suitable numerical values of the parameters involved. But at 77°K their measurements show two peaks whereas according to our model we expect only one peak, since like Ge the optical absorption measurements of Braunstein (1959) in *p*-type GaAs, show only one maximum at 77°K. Thus it is necessary that this point be further investigated.

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